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APPLICATION OF THE HOLOGRAPHIC INTERFEROMETRY METHOD TO DETERMINE THE STRESS INTENSITY FACTOR

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Methods are analyzed for the determination of the stress intensity factor K_I by means of experimentally found displacements in the area of a crack apex. The method of holographic interferometry for recording holograms by the scheme of opposing beams is used to measure the displacements. In order to raise the hologram quality and the accuracy it is recommended to superpose a high-frequency metallized raster on the structure surface. A method is described for finding K_I by opening the crack. Examples are presented of investigation of a calibration specimen and a ribbed panel with a fatigue crack.

When studying structures with cracks the stress intensity factor can be found from experimentally measured displacements in the area of the crack apex [1-3]. Both the displacements u, v in the plane of the specimen [1, 2] and the displacements w out of the plane of the specimen [3] are used to do this. All three displacement vector components can be determined experimentally by using the holographic interferometry method [4]. Two schemes are possible for obtaining the initial information, the hologram recording: an extra-axial [4] and opposing beam [5] scheme. The second scheme is preferable in investigations of real structures or their elements since it permits consolidating the recording medium on the surface of the object being considered, the applied holographic interferometer, which substantially reduces demands for vibration-insulation of both the testing equipment and the optical elements. Moreover, recording of the displacements of the object as a single whole is eliminated, which simplifies processing the interference patterns.

However, in addition to the advantages, such a hologram recording scheme possesses an essential disadvantage, low hologram quality. Holograms can be restored only in the laser light beam used for the recording, the interference fringe patterns are interferograms that are observed only in beams reflected from the holograms. The former circumstance results in the appearance of speckles, which makes recording of the interference fringes difficult in areas with high displacement gradients, and the latter results in the displacement w yielding the greatest contribution to the interference fringe formation. This displacement vector

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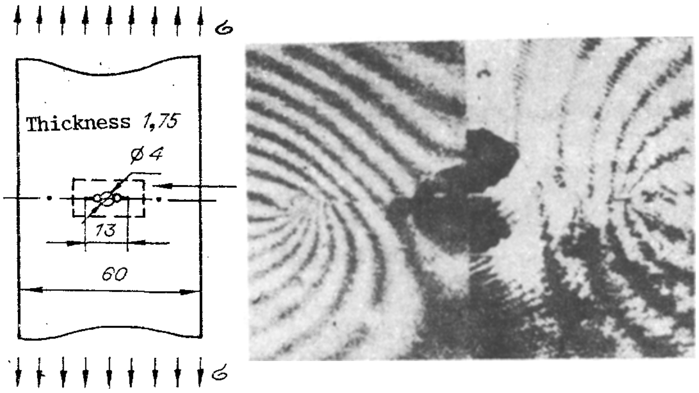

Fig. 1

component is found with high accuracy while the accuracy of determining the u and v components diminishes with the increase in w. At the same time, in the majority of cases the quantity K_I in investigations of real structures can only be calculated from the displacements u or v. This is explained by the fact that the displacement u(v) is determined relative to the crack apex while the displacement w is determined relative to the structure domain removed from the crack where w = const. As a rule it is impossible to extract this domain. Superposition of a high-frequency metallized raster on the object surface permits improvement of the hologram quality and, therefore, reduction of the influence of w and raising the accuracy of finding u and v. The fabrication technology and superposition for such rasters are described in [6]. In this case the holograms obtained by the opposing beam scheme are restored by white noncoherent light, which eliminates the appearance of speckles and the interferograms can be recorded in transmitted light [7].

Two interferograms obtained upon loading a specimen of D16T material with a fatigue crack are presented in Fig. 1 as an example. The interferogram recorded in reflected light for restoration of the hologram by the laser light beam is presented to the left of the crack axis, and in transmitted light for hologram restoration by a white noncoherent light beam on the right. A 910 lines/mm frequency raster was superposed on the specimen. It is seen that the fringe patterns in the interferograms have a qualitative distinction near the crack apex which is explained by the influence of w. Moreover, the resolution of the interferogram obtained in transmitted light is substantially higher.

Therefore, superposition of a metallized raster on the surface of the object under investigation permits determination of u and v at the crack apex even for significant displacements w.

As a rule, v out of the displacements u and v is utilized to calculate K_I since it is greater than u, meaning, it is found with greater accuracy. To determine the displacement v, the hologram is restored under exposure along its normal while the interference fringe patterns are recorded at the angles $\pm\alpha$ to the normal (corresponding to the first diffraction order) so that the recording direction would be perpendicular to the crack. The magnitude of the displacement is determined from the formula [5]

$$v = \frac{1}{2f}(N_1 - N_2), \tag{1}$$

where f is the raster frequency, N_1 and N_2 are the interference fringe orders at the point of determining v at the fringe patterns recorded from the directions $\pm\alpha$. Starting from this, the displacements of the crack edges will be determined most accurately since the points for measuring v on the fringe patterns are easily identified here. The value of K_I is calculated from the formula [8]

$$K_I = \delta \frac{E}{8} \sqrt{\frac{2\pi}{r}}, \tag{2}$$

that is valid for the plane state of stress. Here δ is the mutual displacement of the crack edges, r is the distance between the crack apex and the point for measuring δ, and E is the elastic modulus of the specimen material.

The determination of K_I is realized as follows in practice. Graphs $N_i \sim r$ along the crack edges are constructed from the interference fringe patterns. Then they are subtracted

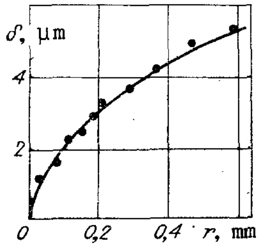

Fig. 2

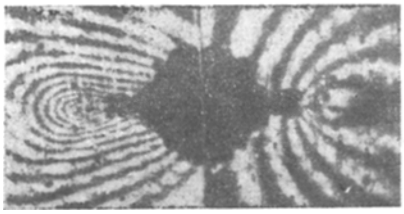

Fig. 3

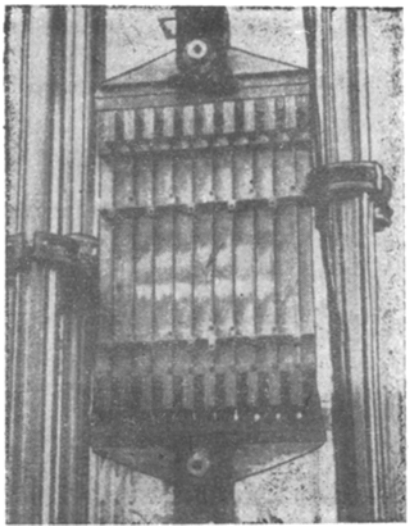

Fig. 4

graphically and the dependence $\delta \sim r$ is constructed by using (1). Chosen from this dependence are 10-15 values of δ in the range $r < 0.05a$ (a is the crack half-length), where (2) is valid [8]. Then the value of K_I is calculated by least square.

The accuracy of the method described above for determining K_I is illustrated in the following examples. A specimen in the shape of a strip with a central crack from the material D16T (see Fig. 1). A 910 lines/mm frequency raster is superposed on one side of the specimen. The holograms were recorded in two exposures for a stress increment of $\sigma_H = 36$ MPa at a distance from the crack. The crack opening was determined at 10 points to the right of the specimen axis for $r < 0.6$ mm from interferograms of the kind represented in Fig. 1. A graph of the dependence $\delta \sim r$ is presented in Fig. 2, the points are experimentally obtained values of δ and the line is the dependence $\delta \sim r$ constructed from (2) after determining K_I. The value of K_I/σ_H was 4.38 $\text{mm}^{1/2}$. The theoretical value for a specimen of this geometry is 4.60 $\text{mm}^{1/2}$ [8]. The difference is within 5% limits.

A monolithic ribbed panel from the D16T material (is shown in Fig. 3). The panel thickness is 3 mm and the stiffness rib 2.5 mm. The distance between the rib axes is $b = 48$ mm and the rib height is 22 mm. The fatigue crack was developed on both sides of a 4 mm diameter hole denoted by the arrow in Fig. 3. The value of K_I was found experimentally for a crack length of $2a = 11$ mm ($a/b = 0.11$). For such an a/b ratio the stiffness ribs have practically no influence on K_I [9] and its value can be computed from the formula

$$K_I = \sigma_H \sqrt{\pi a}. \tag{3}$$

Holograms were recorded on the smooth panel surface with a 800 lines/mm frequency raster superposed. The presence of the stiffness ribs results in the fact that significant displacements out of the plane, buckling of the crack area, occurred during loading of the panel. This is indicated by the ellipsoidal fringes on the interferogram photographed in reflected light for α close to 0 and represented in Fig. 4 to the left of the hole axis. Such displacements reached 8-10 μm. However, this did not affect the form of the interferograms obtained in transmitted light in practice. One of them is presented in Fig. 4 to the right of the hole axis and agrees qualitatively with the interferogram presented in Fig. 1. Displacements of the crack edges are determined from the interferograms and then the crack opening at 10 points at the apex also. Processing them according to (2) yielded the value $K_I/\sigma_H = 4.26$ $\text{mm}^{1/2}$. The theoretical value from (3) is 4.16 $\text{mm}^{1/2}$. Here, as in the first example, the difference does not exceed 5%.

Consequently, the deduction can be made that utilization of an applied holographic interferometer during superposition of high-frequency rasters on an object surface will permit determination of the stress intensity factor for cracks in real structures within 5% limits of accuracy.

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